CHAPTER 3

PHYSICAL CAPITAL

It is not a bad definition of *man* to describe him as a *tool-making animal*. His earliest contrivances to support uncivilized life were tools of the simplest and rudest construction. His latest achievements in the substitution of machinery, not merely for the skill of the human hand, but for the relief of the human intellect, are founded on the use of tools of a still higher order.

—Charles Babbage[1]

The economist's name for tools—the physical objects that extend our ability or do work for us—is **capital.** Capital includes not only the machines that sit in factories, but also the buildings in which we work, infrastructure such as roads and ports, vehicles that we use for transporting goods and raw materials, and even computers on which professors compose textbooks. Performing almost any job requires the use of capital, and for most jobs, the worker who has more or better capital to work with will be able to produce more output.

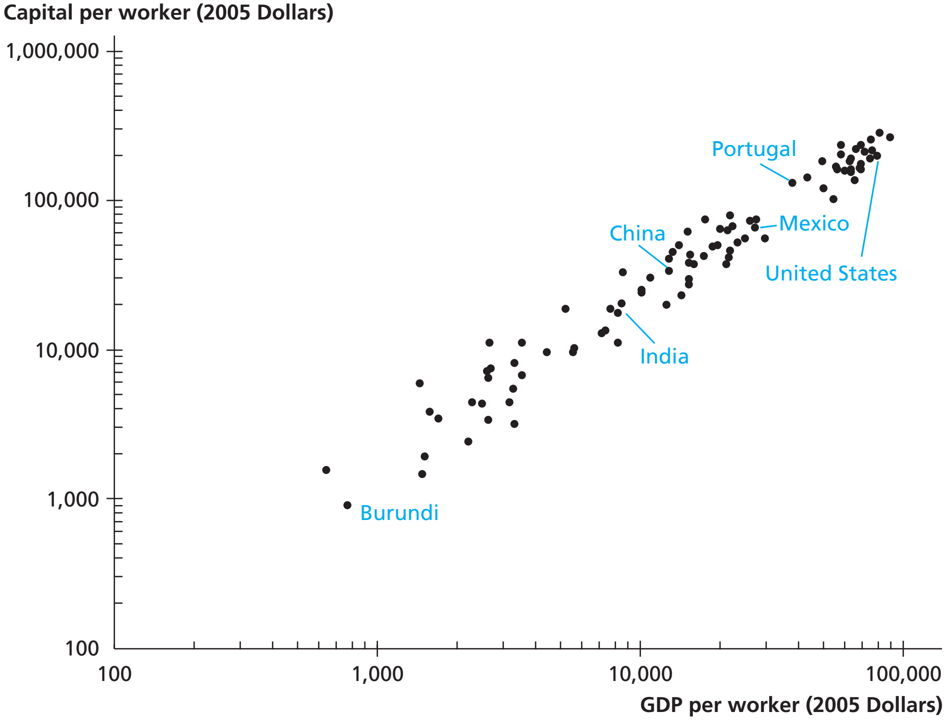
Because workers with more capital can produce more output, differences in the quantity of capital are a natural explanation to consider for the differences we observe in income among countries. In 2009 the average U.S. worker had $201,618 worth of capital to work with. In Mexico in that year, the capital per worker was $66,081, and in India, it was only $17,918.[2] Figure 3.1 looks at the relationship between the amount of capital per worker and the level of GDP per worker for many countries. The close relationship between these two variables is striking. The huge differences in the amount of capital available to workers are an obvious explanation for the large differences in output among these countries. But as the discussion in Chapter 2 made clear, we will need to analyze the problem more carefully before we can conclude that the United States is richer than Mexico or India because it has more capital.

This chapter presents a capital-based theory of why countries differ in their levels of income. Such a simple model cannot explain all of the phenomena we observe, but it is instructive to see how far the model can take us. Many of the concepts introduced in this capital-based model will be useful later on when we

[1]Babbage (1851).
[2]Calculations based on Heston et al. (2010).

FIGURE 3.1

GDP and Capital per Worker, 2009



Source: Calculations based on Heston et al. (2010).

consider further complexities. Examining this model will also give us a chance to apply in a simple environment some mathematical techniques we will use later in the book.

3.1 THE NATURE OF CAPITAL

For the purposes of understanding the capital-based theory of income differences, we need to consider five key characteristics of capital: It is productive; it is produced; its use is limited; it can earn a return; and it wears out. Let's consider each characteristic in more detail.

Capital is productive; using it raises the amount of output that a worker can produce. We explore this property extensively in the next section.

Capital is something that has itself been produced; it has been built or created. The process of producing capital is called **investment.** The fact that capital is produced distinguishes it from a natural resource (such as a piece of land), which also allows a worker to produce more output but is not itself produced. Because it is produced, capital requires the sacrifice of some consumption. That is, the resources used to create a piece of capital could have been used for something else. A modern economy spends a large fraction of output on building new pieces of capital. For example, in 2009 the United States spent $2.1 trillion, or 16.6% of its gross domestic product (GDP), on investment. A country that lowers its investment, for whatever reason, will have more resources left over to spend on consumption.

The decision to build capital might be made privately (in the case of a piece of productive equipment) or by the government (in the case of a piece of infrastructure such as a road). In either case, corresponding to the creation of a piece of capital is an act of investment: the spending of resources on the creation of capital. Investment, in turn, has to correspond to an act of saving. Someone who had control over resources and could have spent them on consumption today has instead used them to build a piece of capital that will be employed in future production.

Capital is *rival* in its use. This is a fancy way of saying that only a limited number of people can use a given piece of capital at one time. In the simplest case—say, a hammer—only one person at a time can use a piece of capital. Other kinds of capital, including roads, can be used by a large but finite number of people at the same time.

Saying that rivalry in its use is one of the characteristics of capital may seem trivial because it is hard to think of many productive tools that *can* be used by an arbitrary number of people at once. However, such tools do exist, in the form of ideas. Like capital, ideas can make a worker more productive. And ideas share with capital the important property that they are the product of investment. In the case of ideas, this investment is called research and development. But ideas differ from capital in that, once an idea is created, an infinite number of people can use it at the same time. (Chapter 8 will discuss this property of ideas at much greater length.)

Because capital is productive and its use is limited, it is often able to earn a return. If using a certain piece of capital will make a worker more productive, then the worker will be willing to pay to use it. In the case of a tool, the worker may act alone to invest in it, buying the tool and then keeping the higher wages earned by using it. In other cases, workers will rent a piece of capital. For example, taxi drivers may rent a cab for a shift. In the case of a more complex economic activity such as building cars, a large quantity of capital (a factory) may be used by thousands of workers. In this case, the workers do not buy or rent capital. Instead, the owners of the capital hire workers, and the profits that remain after the workers are paid are the return to the owners of the capital.

The return that capital earns is often the incentive for its creation. If you decide not to consume some of your income this year and instead invest it in the capital of some corporation, you do so in the hope of earning payments for the use of your capital in future years. Not all capital is privately owned, however. Infrastructure such as roads and ports is usually built and owned by governments.

Finally, capital wears out. The economic term for this wearing-out process is **depreciation.** Using a piece of capital usually causes it to wear down a little. Even when use itself does not cause wear, capital will depreciate simply because of the passage of time: It will rust or rot or get damaged by weather. Depreciation is a routine part of economic life, and no one would buy a piece of capital without taking it into account. A large fraction of the investment that takes place in the economy serves only to replace capital that has depreciated.

3.2 CAPITAL'S ROLE IN PRODUCTION

The first distinguishing characteristic of capital is that it is productive: It enables workers to produce more output. This section examines the relationship between capital and output more formally, to lay the mathematical foundation for a capital-based theory of why countries differ in their levels of income.

Using a Production Function to Analyze Capital's Role

We analyze capital's role in production using the concept of a production function. Recall from Chapter 2 that a production function expresses the relationship between inputs (i.e., factors of production) and the amount of output produced. For simplicity, we consider the case in which there are only two inputs into production: capital, symbolized by K, and labor, symbolized by L. Letting Y symbolize the quantity of output, we can write the following production function:

$$Y = F(K, L).$$

Two assumptions about this production function should be familiar from basic microeconomics. First, we assume that the production function has **constant returns to scale.** In other words, if we multiply the quantities of each input by some factor, the quantity of output will increase by that same factor. For example, if we double the quantity of each input, we will double the quantity of output. Mathematically, this assumption implies that

$$F(zK, zL) = zF(K, L),$$

where z is any positive constant.

Instead of examining the quantity of *total* output in a country, it is frequently more interesting to look at the quantity of output *per worker*.[3] The fact that the production function has constant returns to scale implies that the quantity of

[3]Notice that output per worker and output per capita are not the same thing because not every person in a country is a worker. Our analysis of capital accumulation in this chapter will be conducted in terms of output per worker, even though the data presented in Chapter 1, which motivated our analysis, were in terms of output per capita. If the ratio of workers to total population were the same in every country, then differences among countries in output per worker would be proportional to differences in output per capita. Chapter 5 will discuss how the ratio of workers to total population might differ among countries or might change over time.

output per worker will depend only on the quantity of capital per worker. We see this result by starting with the production function, $Y = F(K, L)$, and then multiplying both inputs by the factor $1/L$:

$$\left(\frac{1}{L}\right)Y = \left(\frac{1}{L}\right)F(K, L) = F\left(\frac{K}{L}, \frac{L}{L}\right) = F\left(\frac{K}{L}, 1\right).$$

The term $1/L$ plays the same role that the constant z played when we defined constant returns to scale.

Defining $k = K/L$ as the quantity of capital per worker and $y = Y/L$ as the quantity of output per worker, we can rewrite this expression as

$$y = F(k, 1).$$

In other words, output per worker is a function only of capital per worker. Finally, because the second term in this per-worker production function does not change, we can ignore this part of the production function and write the per-worker production function as

$$y = f(k).$$

A second assumption about the production function is that it displays diminishing marginal product. The **marginal product** of a particular input is the extra output produced when one more unit of the input is used in production. For example, the marginal product of capital is the increase in output that results from adding one more unit of capital, or equivalently the amount that output per worker rises if one additional unit of capital per worker is used in production. Mathematically, the marginal product of capital (MPK) is given by the following equation:[4]

$$\text{MPK} = f(k + 1) - f(k).$$

The assumption of **diminishing marginal product** says that if we keep adding units of a single input (holding the quantities of any other inputs fixed), then the quantity of new output that each new unit of input produces will be smaller than that added by the previous unit of the input. Figure 3.2 illustrates diminishing marginal product. The horizontal axis shows the quantity of capital per worker, and the vertical axis shows the quantity of output per worker. The marginal product of capital is the slope of this function: the quantity of extra output per worker that results from using one more unit of capital per worker as an input.

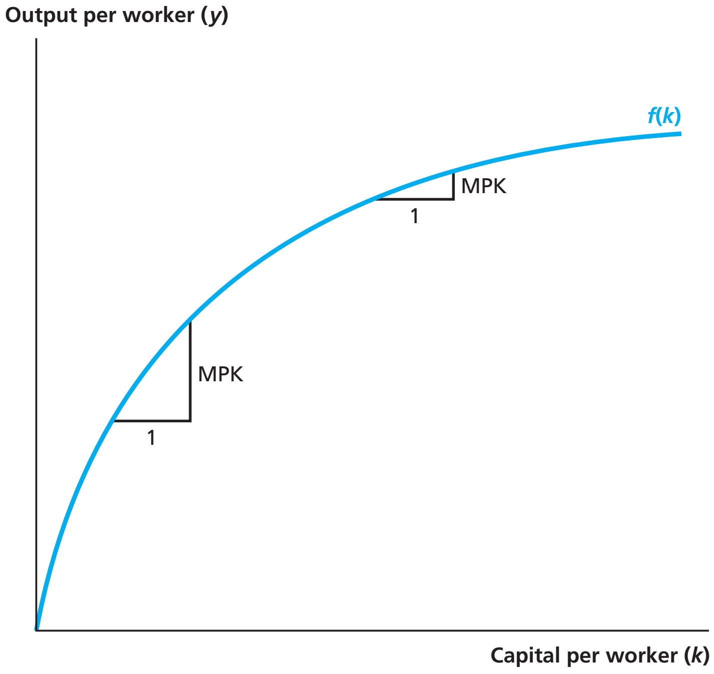
It is often helpful to use a specific functional form for the production function. Throughout this book we will use a **Cobb-Douglas production function.** This production function does a good job of fitting data on inputs and outputs. The Cobb-Douglas production function is

$$F(K, L) = AK^{\alpha}L^{1-\alpha}.$$

[4]Mathematical Note: Using calculus, the marginal product of capital is the derivative of the production function with respect to capital: $MPK = \partial F(K,L)/\partial K$ or, in per-worker terms, $MPK = df(k)/dk$.

FIGURE 3.2

A Production Function with Diminishing Marginal Product of Capital



The parameter A can be thought of as measuring productivity: For given quantities of capital, K, and labor, L, a country with bigger A will produce more output. The parameter α (the Greek letter alpha), which is assumed to have a value between 0 and 1, determines exactly how capital and labor combine to produce output. We will discuss how economists estimate the value of α (see box "Capital's Share of National Income").

To write the Cobb-Douglas production function in per-worker terms, multiply both inputs and output by $1/L$:

$$y = \frac{Y}{L} = \frac{F(K, L)}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{L}\right)^{1-\alpha} = Ak^{\alpha}.$$

Restating the solution, we have an expression for per-worker production:

$$y = Ak^{\alpha}.$$

The appendix to this chapter presents a more detailed mathematical analysis of the Cobb-Douglas production function.

CAPITAL'S SHARE OF NATIONAL INCOME

The share of national income earned by holders of capital is one of the crucial pieces of data that economists examine in studying economic growth. Knowing capital's share of national income will tell us the value of the key parameter α if the production function is of the Cobb-Douglas form.

Figure 3.3 shows data on capital's share of national income for a sample of 53 countries.* The average share in this sample is 0.35, or almost exactly one-third. Most countries' capital shares lie fairly near this average, although there are some significant exceptions. For example, in Botswana and Ecuador capital's share of income is estimated to be 0.55, whereas in Greece it is estimated to be only 0.21. It is also interesting to note that there is no systematic relationship between capital's share of national income and the level of GDP per capita; countries that are rich do not tend to have either higher or lower capital shares than countries that are poor. (In the United States capital's share of national income has ranged between 0.25 and 0.35 since 1935.)†

There is no good theory to explain why the share of capital in national income differs among countries as shown in Figure 3.3. One distinct possibility is that this is a case of measurement error. It could be that the *true* value of capital's share is the same in every country but that the available measurements contain a good deal of "noise" that makes capital's share appear to vary. A piece of evidence in favor of this theory is that there tends to be much less variation in the measured share of capital in national income among rich countries (which tend to have better data) than among poor countries.

Based on these results, we will use a value of 1/3 as our estimate of α throughout this book. Given the messiness of the data, it is unlikely that this estimate is exactly right, but it can serve as a good approximation.

*Bernanke and Gürkaynak (2002), table 10 and note 18.
†Gollin (2002).

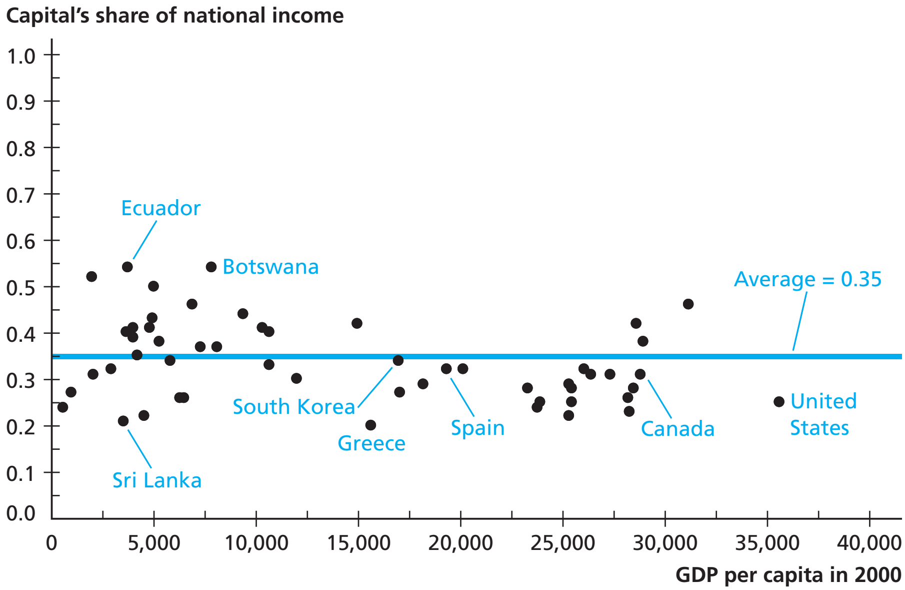
Factor Payments and Factor Shares

Much as the return earned by capital motivates the investment that creates capital, the return earned by labor (i.e., the wage) motivates people to supply their labor to the economy. Later on, in considering other factors of production, we will see that they earn returns as well. Observations about these factor returns often are useful for assigning values to parameters of the production function such as α.

Recall from your previous economics courses that in a competitive economy, factors of production will be paid their marginal products. To see why, consider the problem that a firm faces in deciding how much of a given factor of production to employ. For example, think about a firm that is deciding how many workers it should have on its payroll. Hiring one more worker will produce extra output equal to the marginal product of labor, or MPL (indeed, this is the definition of the marginal product of labor). If the wage were lower than the MPL, the firm

FIGURE 3.3

Capital's Share of Income in a Cross-Section of Countries



Source: Bernanke and Gürkaynak (2002), table 10 and note 18.

would want to hire more workers because each worker would earn the firm more than he or she costs. But because there are diminishing returns to labor, each time a new worker is hired, the MPL will fall, and eventually the MPL will be equal to the wage—at which point the firm would not want to hire any more workers. Similarly, if the wage were above the MPL, then the firm would want to eliminate workers until the MPL and the wage were equal. Thus, in choosing their optimal quantity of labor, firms will set the MPL equal to the wage. Similarly, the marginal product of capital will equal the "rental rate" of capital (i.e., the cost of renting one unit of capital for one unit of time).

In a Cobb-Douglas production function, there is a neat relationship between factor payments and the parameters of the production function. The Cobb-Douglas production function is

$$Y = AK^{\alpha}L^{1-\alpha}.$$

In the appendix, we show that the marginal product of capital for this production function is

$$\text{MPK} = \alpha A K^{\alpha-1} L^{1-\alpha}.$$

In a competitive economy, the marginal product of capital will equal the rental rate per unit of capital—in other words, the amount firms are willing to pay to use a unit of capital. The *total* amount paid out to capital will equal the rental rate per unit of capital multiplied by the total quantity of capital, that is, MPK $\times$ K. **Capital's share of income** is the fraction of national income (Y) that is paid out as rent on capital. Mathematically, the capital share is given by this expression:

$$\text{Capital's share of income} = \frac{\text{MPK} \times K}{Y} = \frac{\alpha A K^{\alpha} L^{1-\alpha}}{A K^{\alpha} L^{1-\alpha}} = \alpha.$$

A similar calculation shows that labor's share is equal to $1 - \alpha$. This result says that even though the quantities of capital and labor in the economy may vary, changes in the rental rate of capital and wage rate will be such that the shares of national income paid out to each factor of production will be unaffected.

This result is important because it tells us we can estimate the value of α just by looking at capital's share of national income. This number is generally estimated to be close to 1/3, and this is the value we will use.

3.3 THE SOLOW MODEL

With a production function that tells us how labor and capital are transformed into output, we can look at a simple model of economic growth that will illustrate the importance of physical capital in explaining differences among countries in their levels of income per capita. The model we examine is called the Solow model, after Nobel Prize–winning economist Robert Solow, who created it in 1956. The Solow model is simple because it focuses on a single dimension along which countries may differ from each other or along which a single country may change over time: namely, the amount of physical capital that each worker has to work with. Because the production function tells us the relationship between capital per worker and output per worker, the only remaining piece of the model to add is a description of how capital per worker is determined.

Determination of Capital per Worker

In this version of the Solow model, we assume that the quantity of labor input, L, is constant over time. We also assume that the production function itself does not change over time; in other words, there is no improvement in productivity. In the case of the Cobb-Douglas production function, this is the same as assuming that the parameter A in the production function is constant. Thus, all of the action in the Solow model comes from the accumulation of capital, which is governed by two forces: investment (the building of new capital) and depreciation (the wearing out of old capital). Later chapters will extend this simple Solow model to allow for changes in the quantity of labor input (Chapter 4), additional factors of production (Chapter 6), and changes in productivity (Part III).

At any point in time, the change in the capital stock is the difference between the amount of investment and the amount of depreciation. If I represents the quantity of investment and D represents the quantity of depreciation, then the change in the capital stock is represented as

$$\Delta K = I - D.$$

Again, it is useful to look at capital accumulation in per-worker terms. Let i and d be the quantities of investment and depreciation per worker. The equation for the accumulation of capital can now be written as follows:

$$\Delta k = i - d.$$

To go further, we must consider how the quantities of investment and depreciation are determined. In the case of investment, we assume that a constant fraction of output is invested. We denote this fraction γ (the Greek letter gamma). This assumption is represented in per-worker terms by the following equation:

$$i = \gamma y.$$

We will return to the question of how investment is determined later in this chapter. For now we treat γ as a constant. In the case of depreciation, we assume that a constant fraction of the capital stock depreciates each period. Denote this fraction δ (the Greek lowercase letter delta):

$$d = \delta k.$$

Combining the three preceding equations, we can write a new equation for the evolution of capital per worker:

$$\Delta k = \gamma y - \delta k.$$

Finally, given that the level of output per worker, y, is a function of the level of capital per worker, k, we can rewrite this equation as

$$\Delta k = \gamma f(k) - \delta k. \tag{3.1}$$

To see how to use this equation, apply it to a concrete example. Suppose that in the year 2010, the quantity of capital per worker in a certain country was equal to 100, the quantity of output per worker—that is, $f(k)$—equaled 50, the fraction of output invested was 20%, and the depreciation rate was 5%. Plug these numbers into the equation:

$$\Delta k = 0.20 \times 50 - 0.05 \times 100 = 10 - 5 = 5.$$

THE RISE AND FALL OF CAPITAL

Our analysis of growth in this chapter is conducted in a setting with only two factors of production: capital and labor. We take this approach both because it is simple—it is easiest to start with two factors and to add more later—and because today capital and labor are the two most important factors of production.

Before the 19th century, however, the most important factor of production other than labor was not capital but land. We can most easily see the changing balance of importance between land and capital by looking at how the value of these two factors has changed. Because both capital and land can be bought and sold, they have an easily observable value. Together, ownership of land and ownership of capital constitute the largest components of wealth. (There are other components of total wealth, such as ownership of houses, gold, or valuables, but these are less important.)

As Table 3.1 shows, the fraction of total wealth held in the form of land has declined dramatically in the United Kingdom over the last three centuries. This decline in land as a fraction of total wealth presumably mirrors a decline in payments to landowners relative to payments to capital owners. This change demonstrates the growing importance of capital as a factor of production.*

TABLE 3.1

Agricultural Land as a Fraction of Total Wealth in the United Kingdom

1688	64%
1798	55%
1885	18%
1927	4%
1958	3%

Why did capital replace land as a key input into production? The most important reason was changes in technology. The Industrial Revolution (beginning in roughly 1760) saw the invention of new technologies, such as the steam engine, that made capital immensely more productive. Similarly, advances in agricultural technology have allowed other inputs, such as chemical fertilizers, to substitute for land. Accompanying these technological changes, there has been a shift in the composition of output, away from food (which requires land to produce) and toward goods that are produced using capital.

Is the rise of capital as a factor of production a permanent feature of economic growth? Not necessarily. Some observers have discerned the rise of a "postindustrial" economy in the most developed countries, where knowledge and skills are taking the place of physical capital as the key inputs into production. If the archetypical laborer of the 1950s worked in a factory filled with big pieces of machinery, the archetype in the 2010s uses no more capital than a laptop computer. In Chapter 6 we introduce the notion of *human capital* as an additional factor of production that includes the skills that are an increasingly important input into production.

Other observers have argued that the reduced importance of land (or natural resources more generally) as the most important factor of production is a temporary phenomenon. In the view of these pessimists, shortages of natural resources will mean that the fraction of national income paid to natural resource holders will rise over time. Chapters 15 and 16 will examine the role of natural resources in production.

*Deane and Cole (1969), Revell (1967).

MEASURING CHANGE OVER TIME

This book is largely concerned with how things change over time—mostly how they grow, but occasionally how they shrink. There are two ways to measure how something changes over time. The first method is to look at how much it changes between one year and the next. We call this measure the **difference,** symbolized with the Greek character Δ (uppercase delta). If x_t is the quantity of something at time t, and x_{t+1} is the quantity at time $t + 1$, then we denote the difference in x between these two periods as Δx_t:

$$\Delta x_t = x_{t+1} - x_t.$$

For example, the population of the United States on July 1, 2009, was 306,656,290; one year later it was 309,050,816. If we use L to symbolize population, then we have

$$\begin{aligned}\Delta L_{2009} &= L_{2010} - L_{2009} \\ &= 309{,}050{,}816 - 306{,}656{,}290 \\ &= 2{,}394{,}526.\end{aligned}$$

In other words, the population increased by slightly less than 3 million people.

It is often more natural to measure how quickly something is changing by looking at its growth rate. The **growth rate** expresses the change in a variable relative to its initial value. Expressed mathematically it is the difference (change over time) divided by the starting value. This book denotes growth rates by putting a "hat" (^) over the variable. Returning to the example of population, we calculate the growth rate as follows:

$$\begin{aligned}\hat{L}_{2009} &= \frac{L_{2010} - L_{2009}}{L_{2009}} \\ &= \frac{2{,}394{,}526}{306{,}656{,}290} \approx .0078 \\ &= 0.78\%.\end{aligned}$$

In other words, population grew by 0.78% over the course of the year. More generally, for any variable x, the difference in x and the growth rate of x are related by this equation: *

$$\hat{x} = \frac{\Delta x}{x}.$$

*Mathematical Note: Readers who know calculus may be familiar with an alternative way of measuring rates of change. Rather than looking at the change in some variable over a discrete amount of time (the difference), we can measure the change in a variable continuously—that is, we can look at the *derivative with respect to time*. This book uses derivatives with respect to time only in some mathematical notes and appendixes. We symbolize these derivatives by putting a dot on top of a variable:

$$\dot{x} = \frac{dx}{dt}.$$

The relationship between the rate of growth and the derivative with respect to time is

$$\hat{x} = \frac{\dot{x}}{x}.$$

Thus, the change in the capital stock per worker was equal to 5 units. The quantity of capital per worker in 2011 was 5 units more than 100, or 105.

Steady States

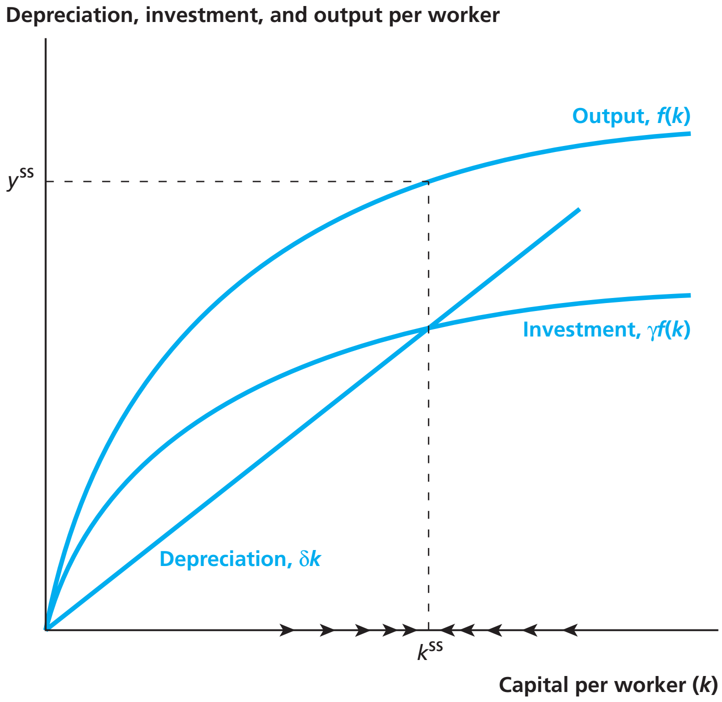
Equation 3.1 describes how capital evolves over time. According to the equation, if investment, $\gamma f(k)$, is larger than depreciation, δk, then the change in the capital stock, Δk, will be positive—that is, the capital stock will be growing. On the other hand, if $\gamma f(k)$ is less than δk, then the capital stock will be shrinking. If $\gamma f(k)$ is equal to δk—in other words, if the quantity of investment equals the quantity of depreciation—then the capital stock will not change at all.

Figure 3.4 analyzes Equation 3.1 graphically. The figure plots the two parts of the right-hand side of the equation—that is, $\gamma f(k)$, which represents investment, and δk, which represents depreciation. To serve as a reference, the figure also plots $f(k)$, the production function. The level of capital at which the lines representing investment and depreciation intersect is called the steady-state capital stock. It is labeled k^{ss} in the figure. If an economy has capital equal to k^{ss}, then the amount of capital per worker will not change over time—thus the name **steady state.**

What if the capital stock is not equal to the steady-state level? The figure shows that over time, the capital stock will move toward the steady state. For example, if the level of capital is below the steady state, then it is clear from the figure that $\gamma f(k)$, the quantity of investment, is greater than δk, the quantity of depreciation. In this case, the capital stock will grow, as we can also see from Equation 3.1.

FIGURE 3.4

The Steady State of the Solow Model



STEADY STATE: A NONECONOMIC EXAMPLE

To help cement the idea of a steady state, consider an example from outside economics: the relationship between the amount of food a person consumes and how much he or she weighs. It is well known that a person who consumes more calories than he or she expends (or "burns") will gain weight, whereas a person who consumes fewer calories than he or she expends will lose weight.

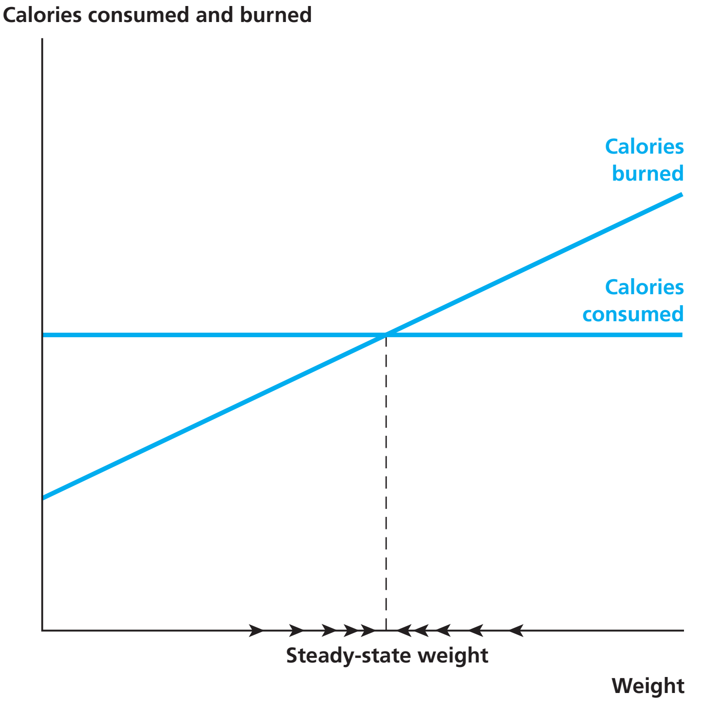
In Figure 3.5, the vertical axis measures a person's daily calorie expenditure and consumption, and the horizontal axis measures the person's weight. We assume that calorie intake does not vary with weight, so calorie consumption is simply shown as a horizontal line. Calorie expenditure, however, rises with weight because a heavier person uses more energy than a light person in the course of daily physical activity. Thus, the line representing calorie expenditure slopes upward.

The figure shows that there will be a steady-state level of weight at the point where these two curves intersect. If a person starts off at less than this weight, calorie intake will exceed usage, and weight will rise. If a person starts off at more than the steady-state weight, calorie intake will be lower than usage, and weight will fall.

Figure 3.5 also shows what factors influence a person's steady-state weight. Raising food intake will shift up the line representing calories consumed and will thus raise steady-state weight. Similarly, a change in lifestyle or environment that causes an upward shift of the curve representing calories burned at any weight will lower steady-state weight.

FIGURE 3.5

Determination of Steady-State Weight



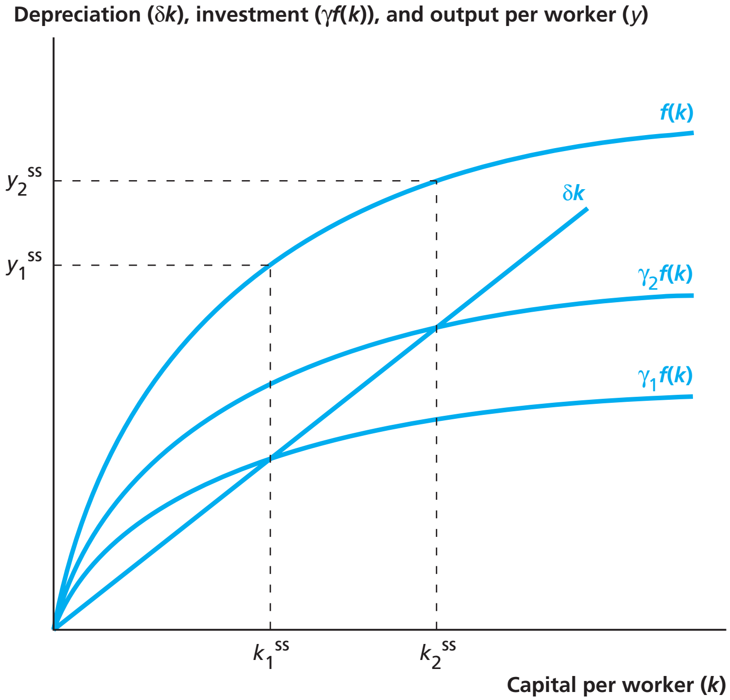
Similarly, if the capital stock is greater than the steady-state level, depreciation will be greater than investment, and the capital stock will shrink over time. The steady state in this case is said to be *stable*: If the economy starts out with any capital stock other than k^{ss}, over time the capital stock will move toward k^{ss}.

Looking again at Figure 3.4, we see that there is a steady-state level of output, y^{ss}, that is associated with steady-state capital stock, k^{ss}. An economy that has capital below k^{ss} will have output below y^{ss}. Similarly, in an economy with any level of output other than y^{ss}, output will move toward y^{ss} over time.

We can also use this diagram to analyze how different aspects of the economy affect the steady-state level of output. Consider a change in γ, the fraction of output invested. Figure 3.6 shows the effect of increasing γ from γ_1 to γ_2. The $\gamma f(k)$ curve shifts upward, as do the steady-state levels of capital and output. Similarly, an increase in δ, the rate of depreciation, would make the δk curve steeper, leading to lower steady-state levels for capital and output.

FIGURE 3.6

Effect of Increasing the Investment Rate on the Steady State



Note: $\gamma_2 > \gamma_1$

Using the Cobb-Douglas production function, $y = Ak^{\alpha}$, we can be more formal in our analysis. Equation 3.1 can be rewritten as

$$\Delta k = \gamma A k^{\alpha} - \delta k. \tag{3.2}$$

Finding the steady state simply entails finding a value of capital, k^{ss}, for which Equation 3.2 is equal to zero,

$$0 = \gamma A (k^{ss})^{\alpha} - \delta k^{ss},$$

which implies that

$$\gamma A (k^{ss})^{\alpha} = \delta k^{ss}.$$

To solve this expression for k^{ss}, first divide both sides by $(k^{ss})^{\alpha}$ and by δ. Then raise both sides to the power $1/(1 - \alpha)$:

$$k^{ss} = \left(\frac{\gamma A}{\delta} \right)^{1/(1-\alpha)}.$$

Plugging this expression for the steady-state level of capital per worker into the production function, we get an expression of the steady-state level of output per worker:

$$y^{ss} = A(k^{ss})^{\alpha} = A^{1/1-\alpha} \left(\frac{\gamma}{\delta} \right)^{\alpha/(1-\alpha)}. \tag{3.3}$$

This equation confirms the result shown in Figure 3.6 that raising the rate of investment will raise the steady-state level of output per worker. Raising γ will raise the numerator of the last term in this equation, so it will raise the steady-state level of output per worker. Similarly, raising the rate of depreciation, δ, will raise the denominator of the same term and will therefore lower y^{ss}.

The Solow Model as a Theory of Income Differences

Equation 3.3 shows how a country's steady-state level of output per worker will depend on its investment rate. If a country has a higher rate of investment, it will have a higher steady-state level of output. Thus, we may think of the Solow model as a *theory of income differences*. Naturally, we should ask how well this theory fits the data. That is, how do actual differences in income among countries compare with the differences predicted by the Solow model?

For simplicity, consider the case where the *only* differences among countries are in their investment rates, γ. We assume that countries have the same levels of productivity, A, and the same rates of depreciation, δ. We also assume that countries are all at their steady-state levels of income per worker, although we will later explore what happens when this assumption is relaxed.

Consider two countries, which we denote i and j. Let γ_i be the rate of investment in Country i and γ_j be the rate of investment in Country j. Their steady-state levels of output per worker are given by the equations

$$y_i^{ss} = A^{1/(1-\alpha)}\left(\frac{\gamma_i}{\delta}\right)^{\alpha/(1-\alpha)}$$

and

$$y_j^{ss} = A^{1/(1-\alpha)}\left(\frac{\gamma_j}{\delta}\right)^{\alpha/(1-\alpha)}.$$

Dividing the first of these equations by the second expresses the ratio of income per worker in Country i to income per worker in Country j:

$$\frac{y_i^{ss}}{y_j^{ss}} = \left(\frac{\gamma_i}{\gamma_j}\right)^{\alpha/(1-\alpha)}.$$

Notice that the terms A and δ have dropped out because both of these parameters were assumed to be the same in the two countries.

We can now make quantitative predictions from our theory. For example, let's suppose that Country i has an investment rate of 20% and Country j has an investment rate of 5%. We use the value of $\alpha = 1/3$, so $\alpha/(1 - \alpha) = 1/2$. Substitute the values of investment into the preceding equation:

$$\frac{y_i^{ss}}{y_j^{ss}} = \left(\frac{0.20}{0.05}\right)^{1/2} = 4^{1/2} = 2.$$

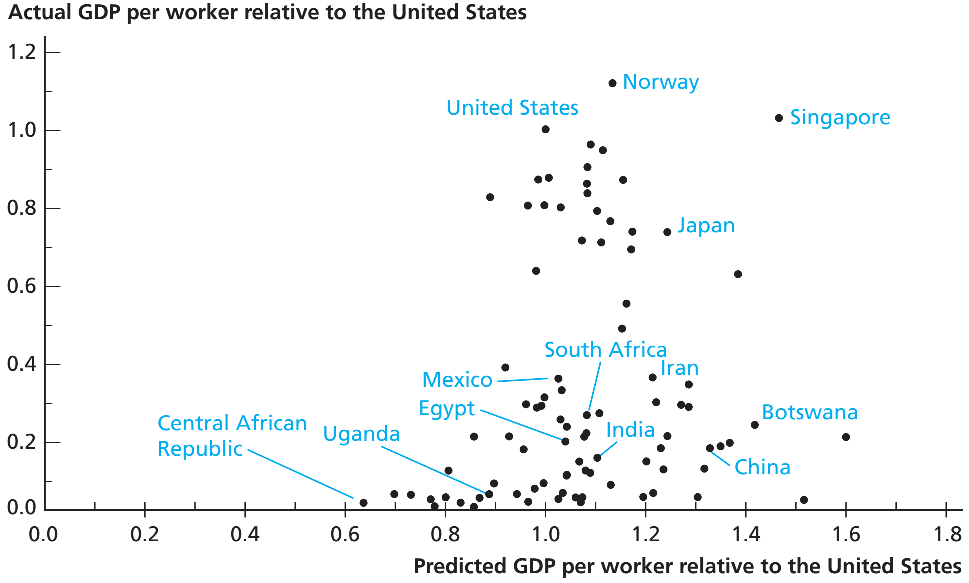
Thus, the Solow model predicts that the level of income per worker in Country i would be twice the level of Country j.

Figure 3.7 shows the results of applying this technique to data from a broad sample of countries. The horizontal axis shows the predicted ratio of income per worker in each country to income per worker in the United States, based on the data on investment rates (specifically, the average ratio of investment to GDP over the period 1975–2009). On the vertical axis are plotted the *actual* ratios of income per worker in each country to income per worker in the United States. If the Solow model worked perfectly, we would expect the data points in Figure 3.7 to lie along a straight line with a slope of 45 degrees; the actual ratio of every country's income per worker to income per worker in the United States would be the same as the ratio predicted by the model. By contrast, if the Solow model had no ability to explain why income differs among countries, no pattern would be visible in this comparison of predicted and actual ratios.

Overall, Figure 3.7 shows that there is some relationship between actual and predicted income, but not a strong one. The correlation of the two series is only 0.17 (although the correlation is 0.35 between the logarithms of the two series).

FIGURE 3.7

Predicted versus Actual GDP per Worker



Source: Author's calculations using data from Heston, Summers, and Aten (2011).

Several countries such as China and Botswana are in the lower right portion of the graph, indicating that they are predicted to be relatively rich but are in fact relatively poor. The United States occupies an unusual position in the figure: according to the model, about half the countries in the sample should have GDP per capita higher than the United States, but in the actual data, only two countries do, Norway and Singapore. Overall, the differences in income among countries that the model predicts tend to be smaller than the actual differences that we observe in the data. For example, the country with the lowest predicted income is the Central African Republic, which is predicted to have 63% of the level of the United States. In the actual data, the figure is 1.9% of the U.S. level.

What are we to make of the imperfect match between the predictions of the Solow model and the actual data on income per worker? For a start, we know there are other influences on countries' income that we have left out of this analysis (or else we would not need the rest of this book!). Specifically, later chapters show how the quantity of capital is determined not only by the level of investment but also by the rate of population growth (Chapter 4), introduce factors of production in addition to physical capital (Chapter 6), and allow for differences in productivity among countries (Chapter 7). Because we have not yet taken account of these factors, we would not expect the model to fit perfectly.

Beyond these reasons for the imperfect fit in Figure 3.7, a previously mentioned reason is that countries might not be in their steady states. Our analysis of the Solow model showed that over time, countries will gradually move *toward* their steady states, not that they will necessarily have reached their steady states at any particular point in time. There are several reasons a country might not be near its steady state at any given time. For example, if part of a country's capital stock were destroyed in a war, it would have a level of capital (and thus output) below its eventual steady state. Similarly, a country might have been at its steady state but then changed its investment rate. The country would then gradually move from its former steady state to its new one, but at the time that we observed it, the country might still be quite far from its new steady state.

In addition to explaining why the Solow model might not fit the data perfectly, the gap between countries' actual levels of income and their steady states can help us use the model to think about differences in the growth rates of income among countries. This is the subject to which we now turn.

The Solow Model as a Theory of Relative Growth Rates

Chapter 1 showed that there are large differences in growth rates among countries. A goal of any growth model should be to explain these differences. Can the Solow model provide an explanation?

The first thing to note is that the Solow model, in the form presented here, will not provide a *complete* explanation of growth rates. The reason is that once a country reaches its steady state, there is no longer any growth! Hence the Solow model will fail to explain growth over long periods of time, during which countries should have reached their steady states. Later in the book we will examine models (some of them extensions of the Solow model) that do explain long-term growth.

Despite this failing of the Solow model, we can still ask whether the model has something to say about *relative* growth rates—that is, why some countries grow faster than others. Here the model has useful predictions.

The key to using the Solow model to examine relative growth rates is to think about countries that are not in the steady state. Because any country that has a constant rate of investment will eventually reach a steady state in which the growth rate of output per worker is zero, all of the growth that we observe in this model will be *transitional*—that is, it will occur during the transition to a steady state. For example, a country with a level of output per worker below the steady state (the result of having a below-steady-state level of capital per worker) will have a growing capital stock and thus a growing level of output. Similarly, a country with output above the steady state will have a falling level of output.

The appendix to this chapter shows that the further below its steady state a country is, the faster it will grow. A country that is far below its steady state will grow very quickly, but as the country approaches the steady state, growth will slow down, approaching zero as the country approaches its steady state. Similarly, if a country has a capital stock far above its steady-state level, its capital stock will shrink rapidly, and this rate of shrinkage will approach zero as the country's capital stock approaches the steady state. We use the term **convergence toward the steady state** to describe this process by which a country's per-worker output will grow or shrink from some initial position toward the steady-state level determined by the investment rate.

Referring to the noneconomic example presented in the box on page 81 may make the intuition for convergence clearer. Consider a man who is currently at his steady-state weight. Suppose he reduces his calorie consumption. This reduction will be represented by a shift downward in the horizontal line in Figure 3.5. The moment calorie consumption falls, the steady-state weight also will fall. But the man's actual weight will not fall right away. Instead, his actual weight will gradually fall because he burns more calories each day than he consumes. As the man's weight falls, however, the number of calories he burns each day also will fall. Thus, the speed with which he loses weight will fall over time, and eventually he will stop losing weight altogether when he reaches the new steady state.

Returning to economic applications, the notion of convergence toward the steady state is the basis for three interesting predictions:

- *If two countries have the same rate of investment but different levels of income, the country with lower income will have higher growth.*

  Because their investment rates are the same, the two countries will have the same steady-state levels of income. If the richer country has income below this steady state, then the poorer country will have income that is even further below the steady state and will grow faster. Conversely, if the poorer country has income that is above the steady state, then the richer country will have income that is even further above the steady state, so the negative effect of moving toward the steady state will be greater in the richer country. Finally, if the poor country has income below the steady state and the rich country has income above the steady state, the movement toward the steady state will have a positive effect on the poor country's growth and a negative effect on the rich country's growth.

- *If two countries have the same level of income but different rates of investment, then the* country *with a higher rate of investment will have higher growth.*

  Of the two countries, the one with a higher rate of investment will have the higher steady-state level of output. If both countries are below their steady states, the country with higher investment will necessarily be *further* below its steady state and so will grow faster. Similarly, if both countries are above their steady states, then the country with low investment will be further above its steady state, so the negative effect on growth of being above steady state will be more pronounced. And if the high-investment country is below its steady state whereas the low-investment country is above its steady state, the high-investment country will grow faster.

- *A country that raises its level of investment will experience an increase in its rate of income growth.*

  If the country was initially at its steady-state level of income, then the increase in investment will raise the steady state. Because income will now be below the steady state, growth will rise. If the country was initially at a level of income below its steady state, the increase in investment will mean it is further below the steady state, so, once again, growth will rise. Finally, if the country was initially at a level of income above its steady state, the increase in investment will mean that income is not as far above steady state or (if the rise in investment is large enough) that income is now at or below the steady state. In any of these cases, the growth rate of income will rise.

These predictions will hold true only if there are no other differences among countries, either in their levels of productivity, A, or in any of the other determinants of steady states that we will consider later in the book. However, the same general pattern of predictions arises from the Solow model when we do account for these other determinants of steady-state income. For example, Chapter 6 will show that the amount of effort that a country devotes to educating its workers functions in much the same manner as the investment rate does in determining the steady-state level of income. Thus, the Solow model predicts that if two countries differ in their levels of spending on education but are similar in other respects (and have equal levels of income), then the country with higher educational spending will grow more quickly. Similarly, the Solow model predicts that a country that suddenly raises its level of spending on education will experience rapid growth as it moves toward its new steady-state level of income.[5]

3.4 THE RELATIONSHIP BETWEEN INVESTMENT AND SAVING

The previous exercises show that the Solow model, though far from perfect, partially answers the questions of why some countries are rich and others are poor, and why some countries grow quickly and others grow slowly. But the answer that the model supplies—that differences in investment rates lead to different steady states—really just pushes the original question back another level. We are left asking why investment rates differ. This is the question to which we now turn.

Previously, this chapter explained that every act of investment corresponds to an act of saving. That is, building capital requires the use of resources that could otherwise have been used for something else. The entity (a person, family, or government) that uses its resources for building capital has forgone the opportunity to consume

[5]For a test of the Solow model's predictions about relative growth rates of countries, see Mankiw, Romer, and Weil (1992).

but in return has become the owner of a productive piece of capital. If we want to ask why investment rates differ among countries, we therefore should think about saving. Perhaps investment rates differ among countries because their saving rates differ. This explanation, however, has a potential problem: Although every act of investment corresponds to an act of saving, it is not true that the amount of investment in a given *country* corresponds to the amount of saving in that country. Why? Because investment can cross national boundaries. For example, a worker in the United States can choose to invest in a piece of capital in Brazil.

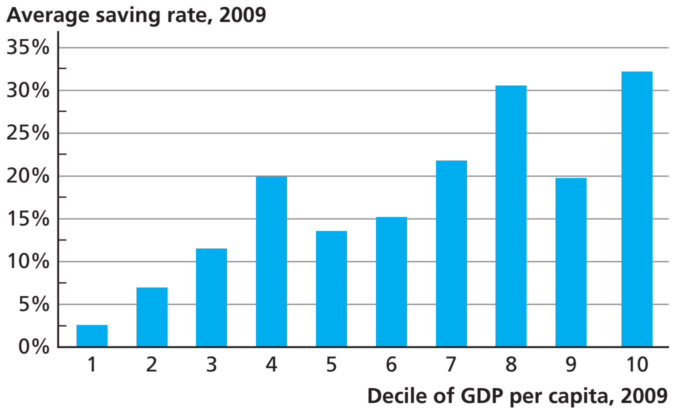
Thus, our investigation of why investment rates differ among countries will have two components. First, we investigate how and why saving rates differ among countries. Second, we explore whether the amount of investment in a country is related to that country's saving, or whether international flows of capital make the amount of saving in a given country irrelevant (or not very relevant) in determining the amount of investment there.

The second of these components—the analysis of flows of investment among countries—is taken up in Chapter 11. There we find that although international flows of investment can be important at times, the most significant determinant of a country's investment rate is indeed its own saving rate. In analyzing saving rates, as we do in the rest of this section, we can think of the saving rate as having the same effect on output, via the Solow model, that the investment rate has.

Figure 3.8 shows the relationship between saving rates and income per capita in a sample of 188 countries. The countries are ranked according to their income per capita in 2009, and the average saving rate is calculated for each decile (the poorest 10%, the next poorest 10%, and so on). The main point from this figure is

FIGURE 3.8

Saving Rate by Decile of Income per Capita



that there is a strong relationship between saving and income per capita. This relationship should not be a surprise, given two other findings: first, the Solow model's prediction that countries with high investment rates have higher levels of income, and second, the finding presented in Chapter 11 that countries' rates of investment are closely related to their rates of saving. So we are left with the question: What determines saving rates?

Explaining the Saving Rate: Exogenous versus Endogenous Factors

Economists distinguish between two types of variables in economic models. **Endogenous variables** are those that are determined within the model. **Exogenous variables** are those that are taken as given when we analyze a model—that is, they are determined outside the model. For example, when we apply the model of supply and demand to the market for bread, the price of bread and the quantity of bread purchased are endogenous variables, whereas factors that shift the supply-and-demand curves, such as the prices of flour and butter, are exogenous variables.

One possible approach to the differences in saving rates among countries is to think of saving as an exogenous variable. Under this interpretation, countries differ in saving rates for reasons that are unrelated to their levels of income per capita. These differences in saving lead to differences in investment rates, which lead in turn, via the Solow model, to differences in the level of income per capita.

If this approach is to help us understand differences in income among countries, however, we need to think about why saving rates differ. In Part IV of this book, we will do just that. There, many of the "fundamental" determinants of economic growth that we will consider have their primary effect on growth by affecting saving rates. Government policy (Chapter 12), income inequality (Chapter 13), culture (Chapter 14), and geography (Chapter 15) will all be examined with an eye toward their possible influence on the saving rate.

Although this approach can take us a long way, it is also important to think about how saving may be affected by income itself. That is, we must consider the possibility that saving is endogenous. Treating the saving rate as an endogenous variable has implications both for how we interpret the data and for how we model growth.

If we allow saving to be an endogenous variable, then the strong relationship between saving rates and income shown in Figure 3.8 is no longer usable as evidence that the Solow model is right. Someone who did not believe in the Solow model (e.g., someone who did not think capital was an important input into production) could argue that most of the relationship between output and saving rates that we observe in the data occurs because saving is endogenous: Countries that are rich save more, but saving more does not make a country rich. This difficulty in interpretation should make us cautious in concluding that the Solow model is the complete explanation for the relationship between saving and growth.

Nonetheless, most economists remain convinced that saving and capital accumulation play a significant role in growth.

Making saving endogenous also has implications for how countries will behave in terms of their growth rates and saving rates. We now examine these implications.

The Effect of Income on Saving

One natural explanation for the low rate of saving in poor countries (as illustrated in Figure 3.8) is that people there simply "can't afford to save." In economic terms, this interpretation says that people in poor countries are living at the margin of subsistence, so they cannot afford to reduce their present consumption to save for the future. Although this argument is plausible for the poorest countries in the world, it fails for even slightly richer countries. If residents of Uganda (average income per capita \$1,152) cannot afford to save because they are on the margin of subsistence, then the same argument cannot be made about the residents of Pakistan (average income per capita \$2,353) because they should be far above the subsistence level.

A variant on this argument focuses not on the constraints that poor people face (i.e., they can not afford to save) but rather on their voluntary choices. The idea is that the decision to save rather than to consume represents a choice between current and future satisfaction, so a person who does not care much about the future will not save. And in turn, according to this theory, being poor makes a person care less about the future. George Orwell nicely summarized this idea when he wrote in *Down and Out in Paris and London* that poverty "annihilates the future."

Whether for these reasons or others, it makes intuitive sense to many people that being poor lowers a person's saving rate, and similarly that poor countries will naturally have lower saving rates than rich countries. What are the implications of this effect for the Solow model? To examine this issue, we will assume that there are no flows of investment among countries, so that in every country, the investment rate is equal to the saving rate. Defining s as the fraction of output that is saved and γ as the fraction of output that is invested, this assumption implies that $s = \gamma$.

The saving rate, in turn, will be taken to depend on the level of income. We first consider the case where saving depends on income in an extreme fashion. Suppose there are two possible rates of saving: s_1, which is low, and s_2, which is high. If income per worker is below some level y^*, then the saving rate will be s_1. If income per worker is greater than or equal to y^*, then the saving rate will be s_2. In the form of an equation,

$$\begin{aligned} \gamma &= s_1 \quad \text{if } y < y^* \\ &= s_2 \quad \text{if } y \geq y^*. \end{aligned}$$

GOVERNMENT POLICY AND THE SAVING RATE

The Solow model provides an explanation for why countries with higher saving rates should have higher levels of income per capita. Government policies that raise the saving rate can thus be a tool to raise the level of national income.

The most direct means by which a government can raise the national saving rate is by using its own budget. The national saving rate has two components: private saving, which is done by households and corporations, and government saving, which is the difference between what the government collects in taxes and what it spends. Budget deficits, which represent negative saving on the part of the government, reduce the national saving rate and thus reduce investment and economic growth.

Governments can also influence the private saving rates by a number of means. One of the most important is in setting up national old-age pension plans. Programs such as Social Security in the United States, in which benefits to the elderly are primarily funded by taxes on those who are currently working, do not generate saving (and thus investment). By contrast, programs in which individuals fund their own retirement by saving during their working years generate a large quantity of capital. During the early 1980s, Chile set up such a "funded" pension system, requiring workers to deposit a fraction of their earnings in an account with a private pension company. Partly as a result of this program, Chile's private saving rate, which had been near zero at the beginning of the 1980s, climbed to 17% by 1991. The success of the Chilean program led Argentina, Bolivia, Colombia, Mexico, Peru, and Uruguay to adopt similar plans in the 1990s.*

A more extreme version of this kind of pro-saving policy was implemented in Singapore. Starting in the 1950s, workers were required to contribute part of their wages to a "central provident fund," which could be used to finance not only retirement but also medical expenditures and the purchase of housing. The government determined the required contribution rate, which reached a high of 40% of a worker's salary in the early 1980s. This forced saving policy was an important determinant of Singapore's phenomenally high saving rate.

Not all pro-saving policies are so coercive, however. The Japanese government, for example, has relied on persuasion to get its citizens to raise their saving rates voluntarily. The government's Campaign to Encourage Diligence and Thrift (1924–1926) featured pro-saving messages on posters on trains and in temples, and in newspaper advertisements, motion pictures, radio broadcasts, and even at rallies. Following World War II, the Central Council for Savings Promotion launched a further series of pro-saving publicity campaigns. Included were programs to educate children about the importance of saving, and the creation of special banks for children within their schools. Japan has had one of the highest saving rates in the world since World War II, although sorting out the extent to which this high saving was as a result of government persuasion is not easy.†

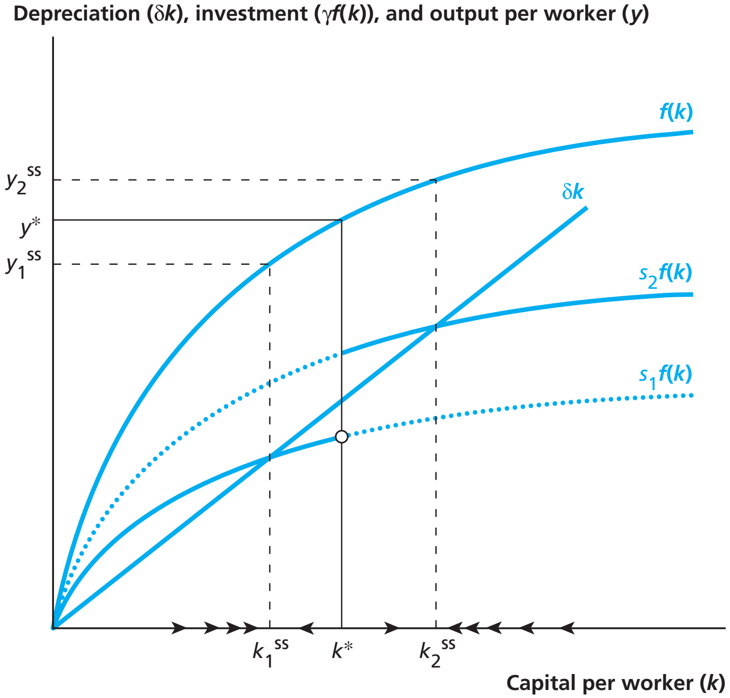
*James (1998).
†Garon (1998).

Figure 3.9 illustrates what can happen in such a situation. It analyzes the same diagram that we used previously for finding the steady state of the Solow model (Figure 3.4). Specifically, it graphs the two terms on the right-hand side of the equation for the change in capital (Equation 3.1):

$$\Delta k = \gamma f(k) - \delta k. \tag{3.1}$$

FIGURE 3.9

Solow Model with Saving Dependent on Income Level



What is new in Figure 3.9 is a jump in the line representing $\gamma f(k)$. To see why, notice that corresponding to the level of income y^* that determines whether a country has a low or high saving rate, there is a level of capital below which saving will be low and above which saving will be high. This level of capital, k^*, can be determined from the production function. If capital is less than k^*, output will be less than y^*, so the saving rate will be s_1. Similarly, if capital is greater than or equal to k^*, output will be greater than or equal to y^* and saving will be s_2.

If the saving rate were always s_1, the steady state of the economy would occur at the level of capital labeled k_1^{ss}; if the saving rate were always s_2, the steady state would occur at the level of capital labeled k_2^{ss}. Notice that the level of capital at which saving switches from low to high, k^*, falls between k_1^{ss} and k_2^{ss}. This means that if the level of capital per worker is below k^*, the saving rate will be s_1, and the economy will move toward the steady state k_1^{ss}. But if the capital stock is above k^*, then the saving rate will be s_2 and the economy will move toward the steady state k_2^{ss}. In other words, there are *two* possible steady states in this economy, and a country will gravitate toward one or the other depending on its initial level of capital.

Figure 3.9 captures the idea that two countries could be completely identical in terms of the underlying determinants of their incomes but still end up with different levels of income per capita in the steady state. A country at the lower steady state can be viewed as being "trapped" there: Its level of income per capita is low because its saving rate is low, and its saving rate is low because its income per capita is low. This is an example of a more general phenomenon known as **multiple steady states,** in which a country's initial position determines which of several possible steady states it will move toward. Economists actively debate the extent to which multiple steady states can explain differences in income among countries. If multiple steady states are important, then differences in income per capita among countries do not necessarily arise because of "fundamental" differences among countries but rather because of self-reinforcing behavior: Being rich leads a country to behave in a manner that keeps it rich, whereas being poor leads a country to behave in a way that keeps it poor.

In Figure 3.9, the dependence of saving on the level of income is quite stark. An alternative story would be that the saving rate rises gradually as the level of income rises, rather than jumping up suddenly at a particular level of income as it does in this figure. In this case, it is still possible that there will be multiple steady states in the economy, but it is also possible that there will be only a single steady state. If there is only one steady state, however, the fact that saving rises with the level of income still has an important implication: The process of convergence toward the steady state will be slow. To see why, consider the case of a country that starts off with income (and thus capital) that is below the steady state. Previously, this chapter showed that in the Solow model with a constant investment rate, such a country will experience rapid growth at first but slower growth as the capital stock nears its steady-state level. In the case in which saving is endogenous, however, a country with income below its steady state will also have a low saving rate, and this low saving rate will reduce the rate of growth. The net result will be that the transitional growth that occurs along the path to the steady state will take place over a longer period of time than it would in the case in which the saving rate was constant.

3.5 CONCLUSION

In this chapter we have examined the role of physical capital in economic growth. The chapter has shown that the Solow model, based around capital accumulation, explains some of the differences in per-worker income across countries and also throws some light on differences among countries' growth rates.

But our analysis pointed out several big deficiencies in the Solow model. As an explanation for differences in income among countries, the model is incomplete. One reason is that it assumes that the only source of differences in income per worker across countries is differences in their per-worker capital stock, ignoring differences in other factors of production or in the production function by which

THE RISE AND FALL OF CAPITAL REVISITED

To classical economists such as David Ricardo (1772–1823) and Thomas Malthus (1766–1834), the most important factor of production other than labor was not capital but land. There was good reason for this focus, for at the time these economists wrote, land was a much more important form of wealth than capital. With the advent of the Industrial Revolution in Europe, however, capital came to play a much more important role in the economy, and economists followed along.

The belief that the accumulation of capital is the key to economic growth reached its high-water mark after World War II. W. Arthur Lewis, who would later win the Nobel Prize, wrote in 1954, "The central problem of the theory of economic development is to understand the process by which a community that was saving and investing 4 to 5 percent of its national income…converts itself into an economy where voluntary saving is running at about 12 to 13 percent. This is the central problem because the central fact of economic development is rapid capital accumulation."* Prominent economist W. W. Rostow, in his influential description of the stages of economic growth, similarly defined an increase in the investment rate as a necessary part of the "take off" to sustained growth.

The apparent economic success of the Soviet Union (which we now know to have been something of an illusion) also contributed to the view that capital accumulation was the key to economic growth. In his famous economics textbook, Paul Samuelson, although noting the inefficiencies of the Soviet system, argued that it would nonetheless succeed in producing growth simply because of the "decision to cut down ruthlessly on current consumption in order to enlarge the flow of capital formation and economic development."

Economists' views on capital's role in producing growth in turn influenced the policies that developing countries and international agencies followed in attempting to promote economic development. In the decades after World War II, developing countries were advised to focus on raising their investment rates, and international aid was targeted toward helping poor countries acquire more capital.

These policies are now largely viewed as having failed. In almost all cases, injections of capital failed to produce significant growth in developing countries. The former Soviet Union itself, with its rusting, useless factories, has provided one of the most persuasive counterarguments to economists who focus exclusively on capital accumulation. Between 1960 and 1989, the Soviet Union devoted 29% of GDP to investment; in the United States, the comparable figure was 21%. Postmortem analyses of the Soviet Union have come to the conclusion that the massive accumulation of capital was accompanied by almost no growth in productivity—and that this failure of productivity growth doomed the economy to eventual stagnation.

In recent decades economists have discarded the view of development with capital accumulation as its centerpiece. They have paid more attention to factors such as education, technological change, and the structure of economic institutions. The downgrading of capital from its central position in development thinking does not mean that it is not important; rather, economists now see capital accumulation as just one of many aspects of economic growth.†

*Lewis (1954).

†Easterly and Fischer (1995), King and Levine (1994).

these factors are combined. Also, even restricting our focus to differences in capital among countries, the Solow model tells us that differences in investment rates are important but does not say anything about the source of these differences in investment rates. Still another drawback of the model is that it does not model long-run growth because in the steady state of the model, countries do not grow at all.

Later chapters will address all of these problems. These chapters will expand the Solow model to accommodate additional factors of production, differences in the production function among countries, and technological change over time. The discussion will draw on many of the ideas established using this simple version of the Solow model, such as convergence toward a steady state.

KEY TERMS

capital **68**
investment **69**
depreciation **71**
constant returns to scale **71**
marginal product **72**
diminishing marginal product **72**
Cobb-Douglas production function **72**
capital's share of income **76**
difference (in a variable) **79**
growth rate **79**
steady state **80**
convergence toward the steady state **87**
endogenous variable **90**
exogenous variable **90**
multiple steady states **94**

QUESTIONS FOR REVIEW

1. Why is capital a natural suspect when we consider differences in income per capita among countries?
2. What is the evidence on the share of income from capital in total national income? What is the common estimate of the share of capital in national income and why is it used?
3. What is the steady state of the Solow model? How does an economy's capital stock change when it is at the steady state?
4. Describe the concept of "convergence" towards the steady state. What does the theory of convergence predict about differences in average per capita income across countries?
5. Why can a country not grow forever solely by accumulating more capital?

PROBLEMS

1. Explain whether or not each of the following is physical capital:
   a. Wheat
   b. A U.S. air force fighter jet
   c. A shopping mall complex
   d. Royalties from the sale of music CDs
2. A country is described by the Solow model, with a production function of $y = k^{1/2}$. Suppose that k

is equal to 900. The fraction of output invested in 50%. The depreciation rate is 10%. Is the country at its steady-state level of output per worker, above the steady state, or below the steady state? Show how you reached your conclusion.

3. Describe in words and with a diagram an example of a steady state from outside of economics, similar to the one discussed in the box on page 81.

4. In Country 1 the rate of investment is 10%, and in Country 2 it is 20%. The two countries have the same levels of productivity, A, and the same rate of depreciation, δ. Assuming that the value of α is 1/3, what is the ratio of steady-state output per worker in Country 1 to steady-state output per worker in Country 2? What would the ratio be if the value of α were 1/2?

5. The following tables show data on investment rates and output per worker for three pairs of countries. For each country pair, calculate the ratio of GDP per worker in steady state that is predicted by the Solow model, assuming that all countries have the same values of A and δ and that the value of α is 1/3. Then calculate the actual ratio of GDP per worker for each pair of countries. For which pairs of countries does the Solow model do a good job of predicting relative income? For which pairs does the Solow model do a poor job?

a.

Country	Investment Rate (Average 1975–2009)	Output per Worker in 2009
Thailand	35.2%	\$13,279
Bolivia	12.6%	\$8,202

b.

Country	Investment Rate (Average 1975–2009)	Output per Worker in 2009
Nigeria	6.4%	\$6,064
Turkey	16.3%	\$29,699

c.

Country	Investment Rate (Average 1975–2009)	Output per Worker in 2009
Japan	29.9%	\$57,929
New Zealand	18.6%	\$49,837

6. Country X and Country Y have the same level of output per worker. They also have the same values for the rate of depreciation, δ, and the measure of productivity, A. In Country X output per worker is growing, whereas in Country Y it is falling. What can you say about the two countries' rates of investment?

7. In a country the production function is $y = k^{1/2}$. The fraction of output invested, γ, is 0.25. The depreciation rate, δ, is 0.05.

   a. What are the steady-state levels of capital per worker, k, and output per worker, y?

b. In year 1, the level of capital per worker is 16. In a table such as the following one, show how capital and output change over time (the beginning is filled in as a demonstration). Continue this table up to year 8.

Year	Capital k	Output $y = k^{1/2}$	Investment γy	Depreciation δk	Change in Capital Stock $\gamma y - \delta k$
1	16	4	1	0.8	0.2
2	16.2				

c. Calculate the growth rate of output between years 1 and 2.

d. Calculate the growth rate of output between years 7 and 8.

e. Comparing your answers from parts c and d, what can you conclude about the speed of output growth as a country approaches its steady state?

8. Consider an economy in which the amount of investment is equal to the amount of saving (i.e., the economy is closed to international flows of capital). Any output that is not saved is consumed. The production function is $y = Ak^{\alpha}$. Find the value of γ, the fraction of income that is invested, that will maximize the steady-state level of consumption per worker. (This is called the "golden rule" level of investment.)

9. In a country, output is produced with labor and physical capital. The production function in per-worker terms is $y = k^{1/2}$. The depreciation rate is 2%. The investment rate (γ) is determined as follows:

$$\gamma = 0.20 \quad \text{if} \quad y \leq 10$$
$$\gamma = 0.40 \quad \text{if} \quad y > 10$$

Draw a diagram showing the steady state(s) of this model. Calculate the values of any steady state levels of k and y. Also, indicate on the diagram and describe briefly in words how the levels of y and k behave outside of the steady state. Comment briefly on the stability of the steady state(s).

For additional exploration and practice using the Online Data Plotter and data sets, please visit www.pearsoninternationaleditions.com/weil.